The Lindemann problem

Reference papers: Richardson W., Volk J., Lau K.H., Lin S.H., Eyring H. (1973). Application of the singular perturbation method to reaction kinetics, *Proc. Natl. Acad. Sci. USA 70*, 1588-1592. Goussis D.A. and Valorani M. (2006). An Efficient Iterative Algorithm for the Approximation of the Fast and Slow Dynamics of Stiff Systems, *J. Comp. Physics 214*, 316-346.

The governing equations in dimensionless form are:

$$\frac{dy}{dt} = \frac{z}{\epsilon}(z-y) - y \qquad \frac{dz}{dt} = -\frac{z}{\epsilon}(z-y) \tag{1}$$

where $\epsilon << 1$. The solution for $\epsilon = 10^{-3}$ is shown in Figure 1.

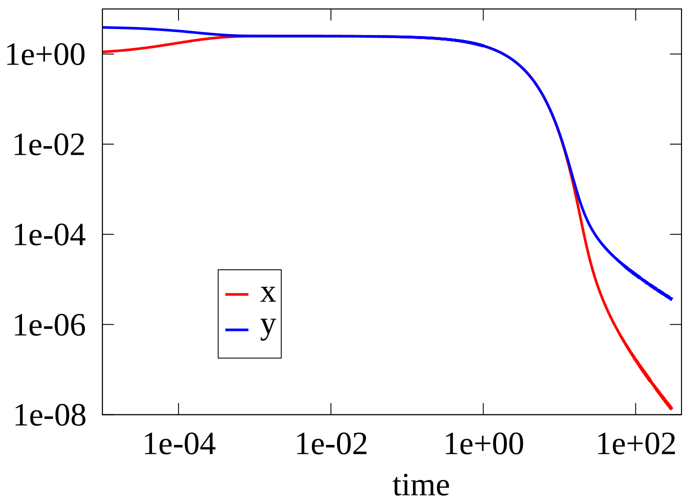

Fig. 1. The evolution of y and z in time; $\epsilon = 10^{-3}$

This model has a non-hyperbolic fixed point, and features a slow manifold with a region of weaker attractivity. In particular, when y and z are greater than $O(\epsilon)$ the manifold is approximated by:

$$y \approx z + \frac{\epsilon}{2} \tag{2}$$

while when y and z are smaller than $O(\epsilon)$ the manifold is approximated by:

$$\epsilon y \approx z^2 + \frac{z^3}{\epsilon} \tag{3}$$

When y and z are $O(\epsilon)$, a suitable transformation yields:

$$\frac{dY}{dt} = Z(Z-Y) - Y \qquad \frac{dZ}{dt} = -Z(Z-Y) \tag{4}$$

where $y = \epsilon Y$ and $z = \epsilon Z$. In this form, it is clear that no time scale gap is generated in this case.

This feature is confirmed in Figure 2, where the evolution of the two fast time scales is presented. In particular it is shown that a time scale gap develops in the case where y and z are either greater or smaller than $O(\epsilon)$; this gap diminishing when y and z are $O(\epsilon)$.

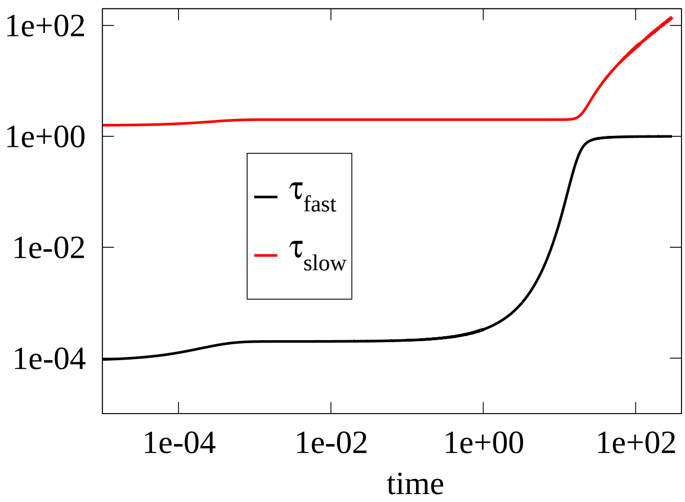

Fig. 2. The evolution of the two time scales in time; $\epsilon = 10^{-3}$

The PDE version of the governing equations in dimensionless form are:

$$\frac{dy}{dt} = \frac{z}{\epsilon}(z-y) - y + \kappa_y \frac{d^2y}{dx^2} \qquad \frac{dz}{dt} = -\frac{z}{\epsilon}(z-y) + \kappa_z \frac{d^2z}{dx^2} \tag{5}$$

where κ_y and κ_z are O(1) constants.